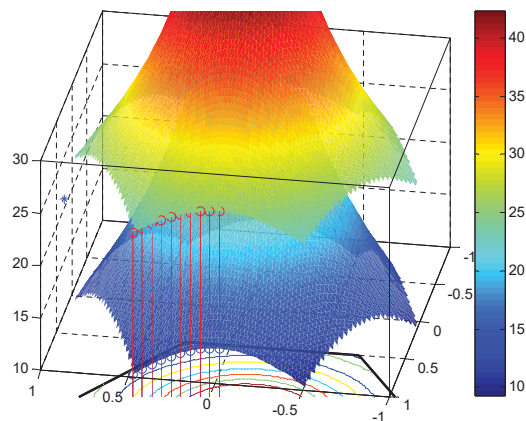


ECS455 Chapter 2

Cellular Systems

2.2 Co-Channel Interference



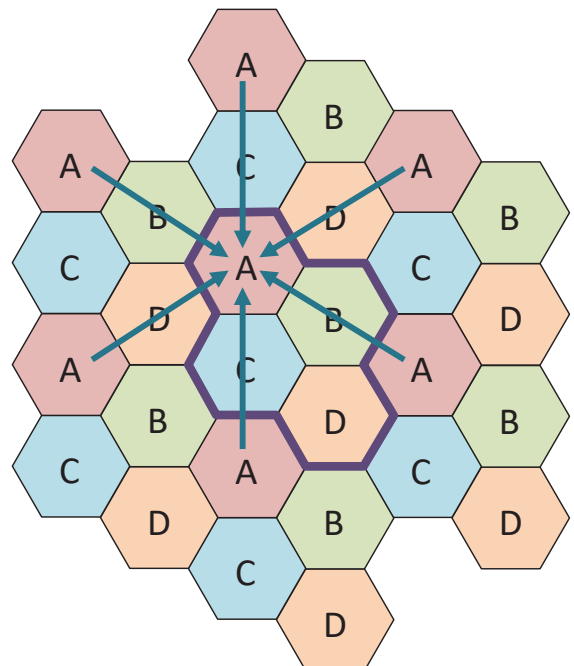
Dr. Prapun Suksompong
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1

(Intercell)

Co-Channel Interference

- Frequency reuse \rightarrow co-channel interference
- Consider only nearby interferers.
 - Power decreases rapidly as the distance increases.
- In a **fully equipped hexagonal-shaped** cellular system, there are always $K = 6$ cochannel interfering cells in the **first tier**.



2

Three Measures of Signal Quality

- For **noise-limited** systems, $\text{SNR} = \frac{P_r}{P_{\text{noise}}}$
Signal-to-noise (power) ratio
- Consider both noise & interference: $\text{SINR} = \frac{P_r}{P_{\text{interference}} + P_{\text{noise}}}$
Signal-to-interference-plus-noise (power) ratio
- The best cellular system design places users that share the same channel at a separation distance (as close as possible) where the intercell interference is just below the maximum tolerable level for the required data rate and BER.
- Good cellular system designs are **interference-limited**, meaning that the interference power is much larger than the noise power.

$$\text{SIR} = \frac{P_r}{P_{\text{interference}} + \text{O}}$$

Signal-to-interference (power) ratio

3

“Reliable” vs. “tolerable”?

(Why not as far as possible?)

Co-channel cells, must be spaced **far enough** apart so that interference between users in co-channel cells does not degrade **signal quality** below **tolerable** levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least **sixty times** higher than the noise/interference power.

[Klemens, 2010, p 54]

$$10 \log_{10} 60 = 17.78 \approx 18 \text{ dB}$$

We will soon revisit and use these numbers for some more specific calculations

4

Review: Simplified Path Loss Model

$$\frac{P_r}{P_t} = \beta \left(\frac{d_0}{d} \right)^\gamma \quad \rightarrow \quad P_r = \frac{P_t \beta d_0^\gamma}{d^\gamma} = \frac{k}{d^\gamma} \propto \frac{1}{d^\gamma}$$

Capture the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- β is a unitless constant which depends on the antenna characteristics and the average channel attenuation
- d_0 is a reference distance for the antenna far-field
 - Typically 1-10 m indoors and 10-100 m outdoors.
- γ is the **path loss exponent**.
 - 2 in free-space model
 - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

[Goldsmith, 2005, Table 2.2]

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SIR (S/I): Definition/Calculation

Caution: Not the same as the K used in Section 1.3

- $K = \#$ co-channel interfering cells
- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a forward channel can be expressed as

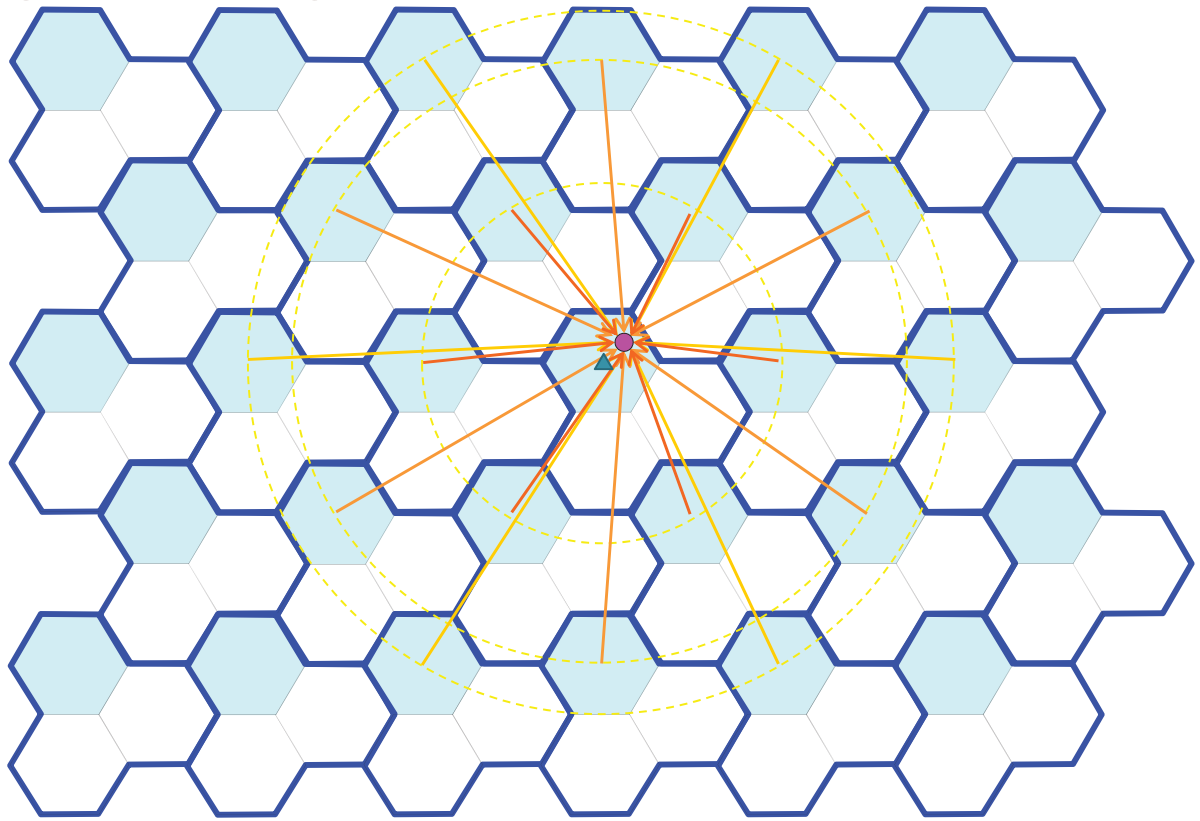
$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^K P_{\text{of the } i^{\text{th}} \text{ interferer}}}$$

- P_r = the desired signal **power** from the desired base station
- P_i = the interference **power** caused by the i th interfering co-channel cell base station.
- Often called the **carrier-to-interference ratio**: CIR.

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[Rappaport, 2002]

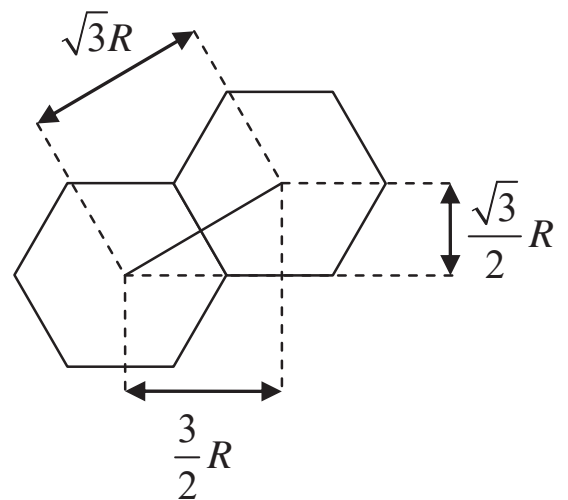
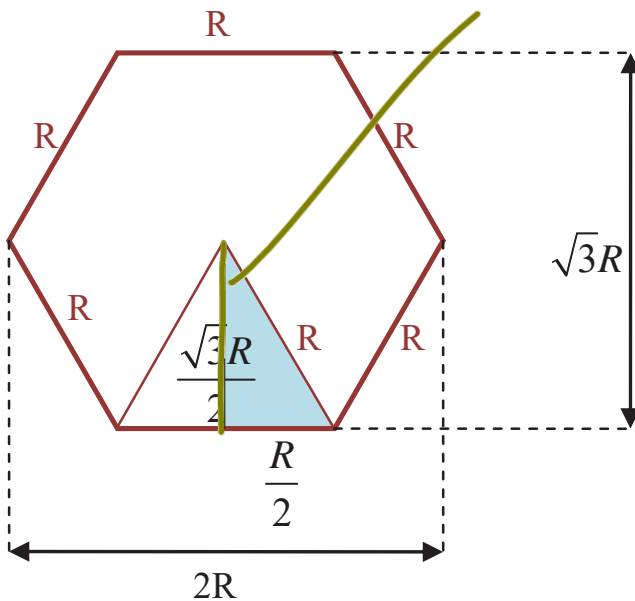
SIR: $N = 3$



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Hexagon

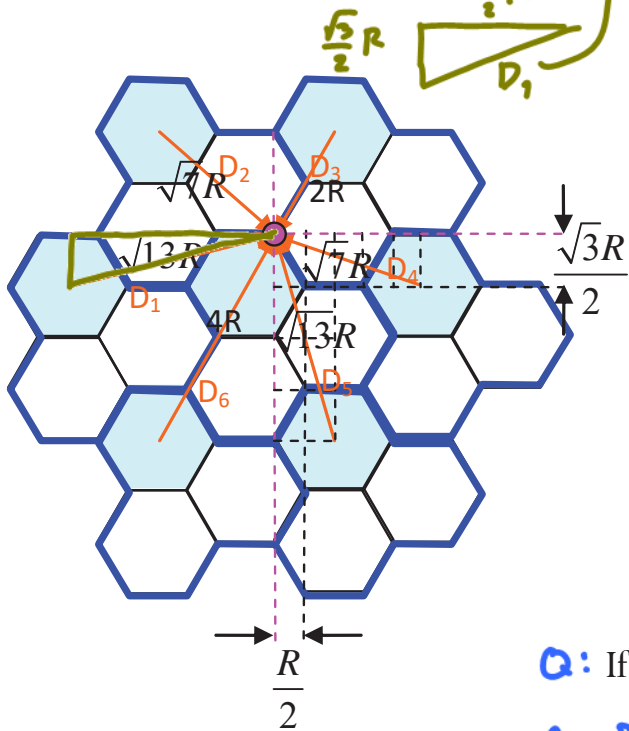
$$\sqrt{R^2 - \left(\frac{R}{2}\right)^2} = R \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} R$$



$$\text{Area} = 6 \times 2 \times \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} R \times \frac{1}{2} R \right) = \frac{3\sqrt{3}}{2} R^2 \approx 2.598 R^2$$

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SIR: $N = 3$



$$D_1^2 = \left(\frac{7}{4}\right)R^2 + \frac{3}{4}R^2 = 13R^2$$

$$D_1 = \sqrt{13}R$$

(Ignore co-channel cells that are too far away)

- Consider only cells in first tier.
- Worse-case distance

$$\begin{aligned} \text{SIR} &\approx \frac{k/R^\gamma}{\sum_i k/D_i^\gamma} = \frac{1}{\sum_i 1/\left(\frac{D_i}{R}\right)^\gamma} = \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \\ &= \frac{1}{2(\sqrt{7})^{-\gamma} + 2(\sqrt{13})^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}} \end{aligned}$$

Q: If $N = 19$, will the SIR be better or worse?

A: Better (larger)

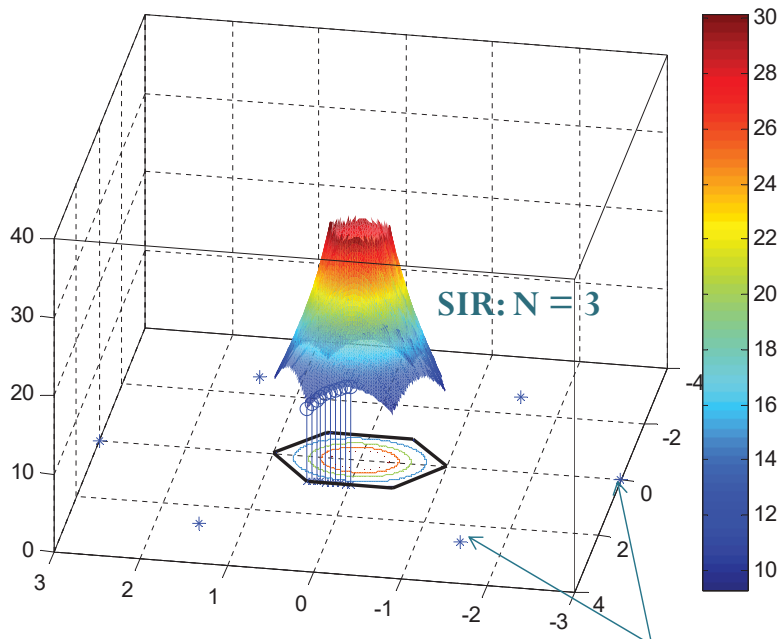
Larger cluster size \Rightarrow greater distances among the cochannel cells.

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SIR: $N = 3$

d = distance between MS and BS

$$\text{SIR} \approx \frac{k/d^\gamma}{\sum_i k/D_i^\gamma} = \frac{1}{\sum_i 1/\left(\frac{D_i}{d}\right)^\gamma} = \frac{1}{\sum_i \left(\frac{D_i}{d}\right)^{-\gamma}}$$

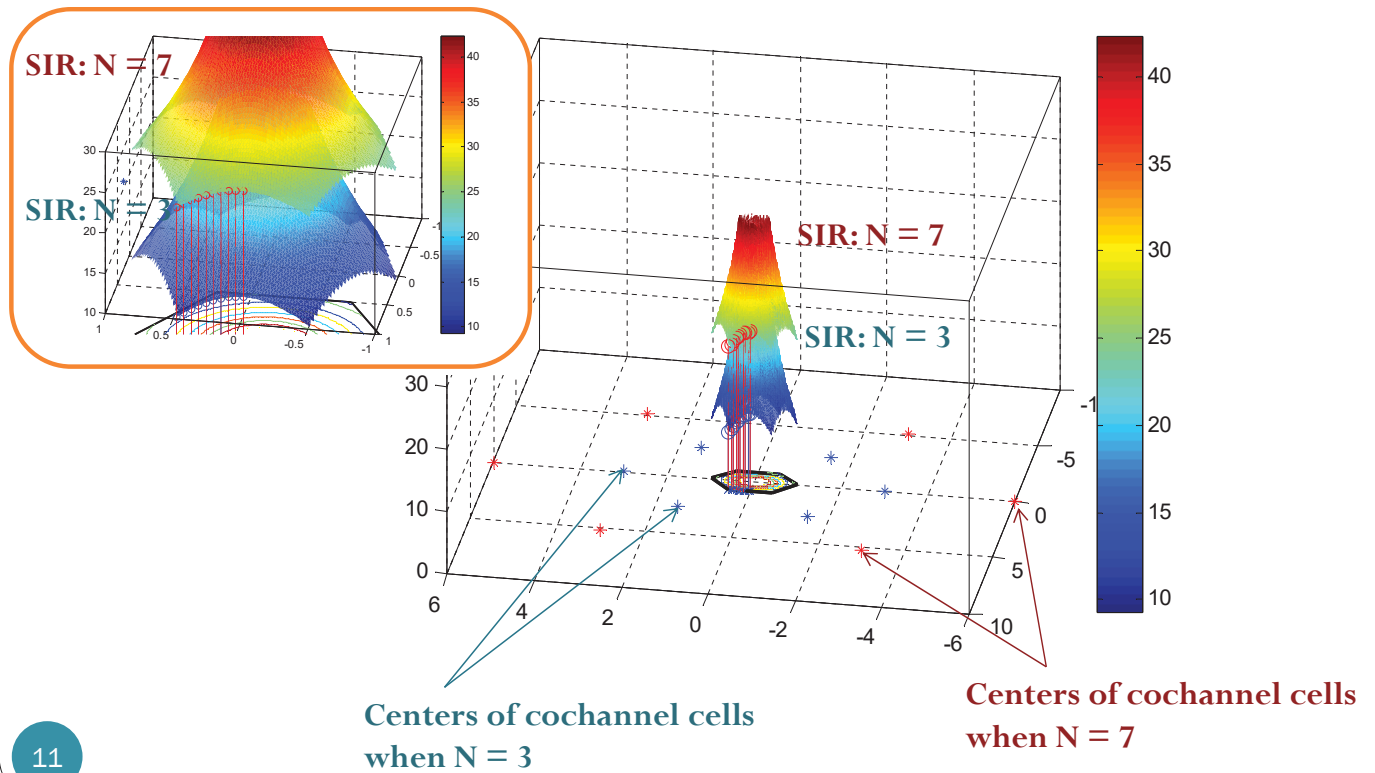


Observe that the SIR value is **smallest** when MS is at any of the **corners** of the hexagonal cell. At such locations, $d = R$ (the cell radius).

Centers of cochannel cells when $N = 3$

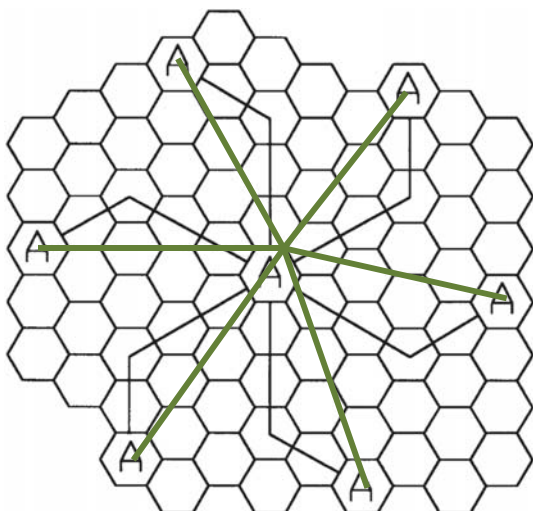
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SIR: N = 3 vs. N = 7



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Approximation



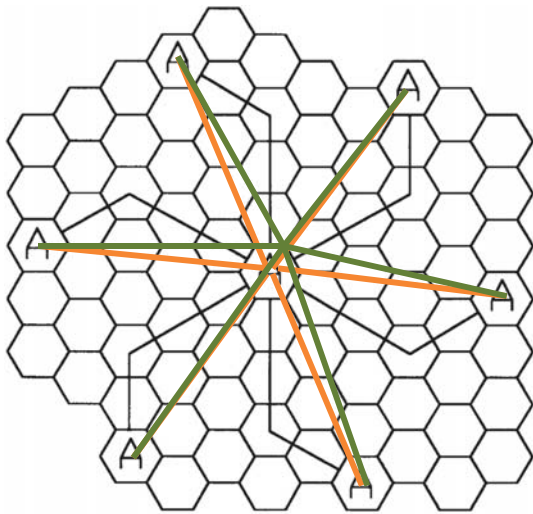
- Consider only first tier.
- Worse-case distance

$$SIR \approx \frac{1}{\sum_i \left(\frac{D_i}{R} \right)^{-\gamma}}$$

- Use the same D for D_i

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Approximation



- Consider only first tier.
- Worse-case distance

$$SIR \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

- Use the same D for D_i

$$SIR \approx \frac{1}{\sum_i \left(\frac{D}{R}\right)^{-\gamma}} \approx \frac{1}{K \left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K} \left(\frac{D}{R}\right)^{\gamma}$$

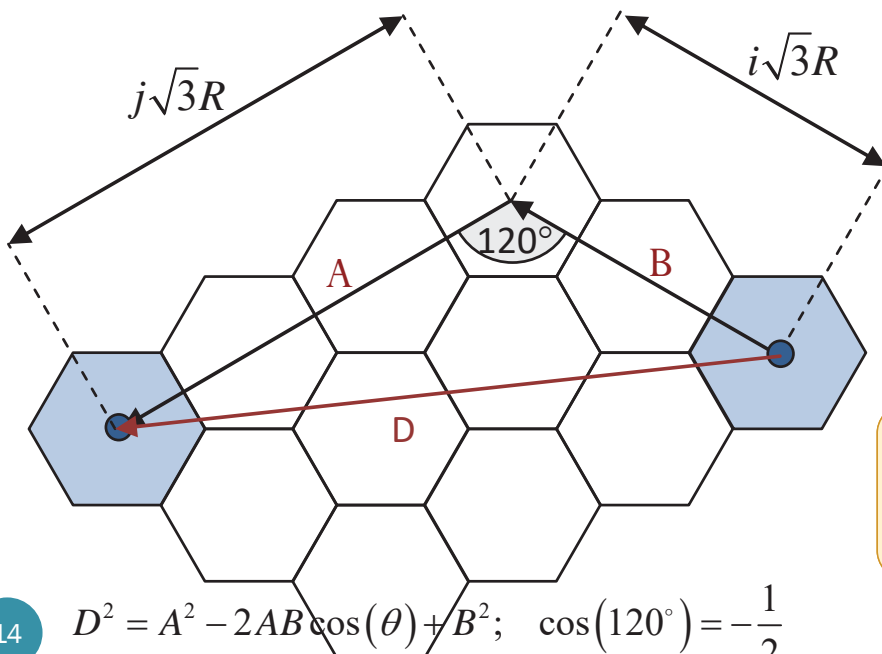
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Notice that D/R is an important quantity!

Center-to-center distance (D)

$$D = \sqrt{(i\sqrt{3}R)^2 + (j\sqrt{3}R)^2 - 2(i\sqrt{3}R)(j\sqrt{3}R)\cos(120^\circ)}$$

$$= R\sqrt{3(i^2 + j^2 + ij)} = R\sqrt{3N}$$



This distance, D , is called **reuse distance**.

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

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$$D^2 = A^2 - 2AB\cos(\theta) + B^2; \quad \cos(120^\circ) = -\frac{1}{2}$$

Q and N

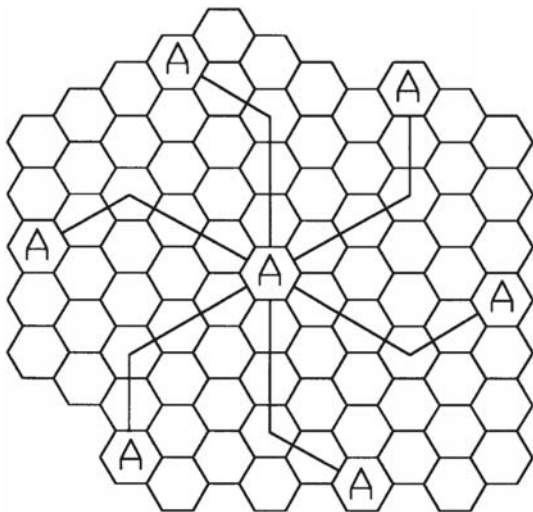
Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

	Cluster Size (N)	Co-channel Reuse Ratio (Q)
$i = 1, j = 1$	3	3
$i = 1, j = 2$	7	4.58
$i = 0, j = 3$	9	5.20
$i = 2, j = 2$	12	6

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Approximation: Crude formula



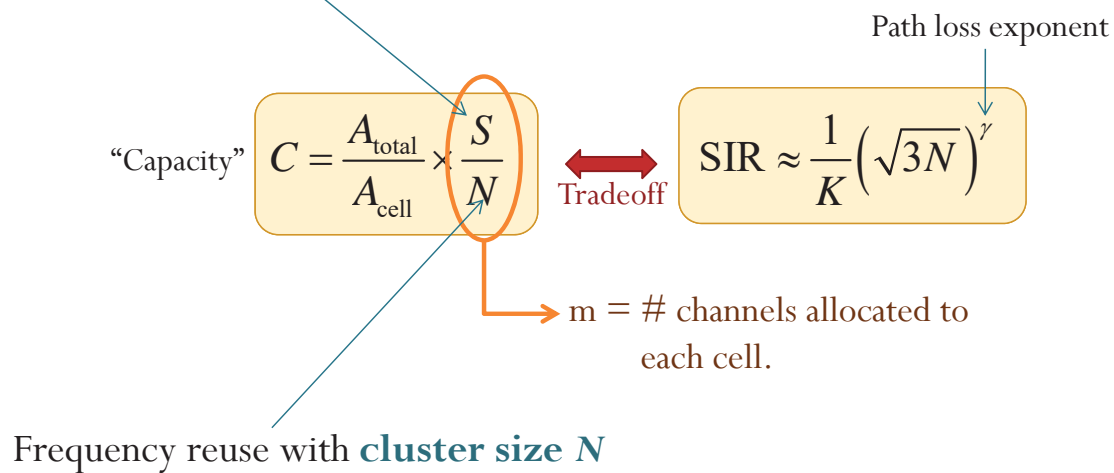
$$\begin{aligned} \text{SIR} &= \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^K P_{\text{of the } i^{\text{th}} \text{ interferer}}} \\ &\approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \approx \frac{1}{K \left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K} \left(\frac{D}{R}\right)^{\gamma} \\ &= \frac{1}{K} \left(\sqrt{3N}\right)^{\gamma} \end{aligned}$$

As the cell cluster size (N) increases, the spacing (D) between interfering cells increases, reducing the interference.

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Summary: Quantity vs. Quality

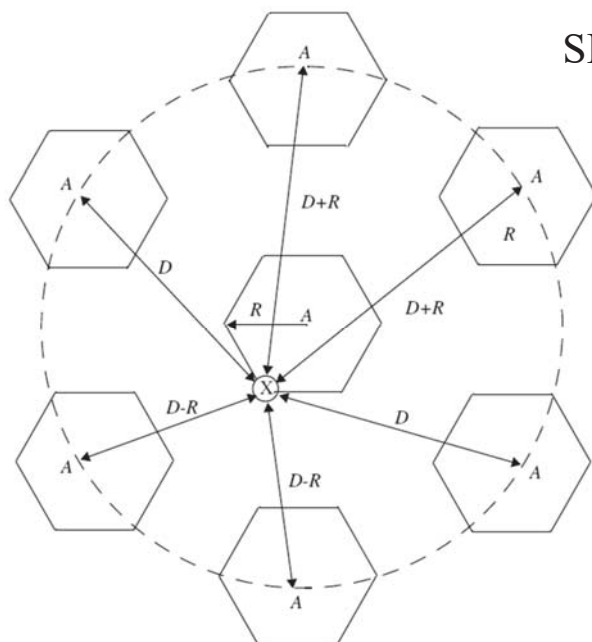
S = total # available duplex radio channels for the system



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SIR: $N = 7$

Better approximation...



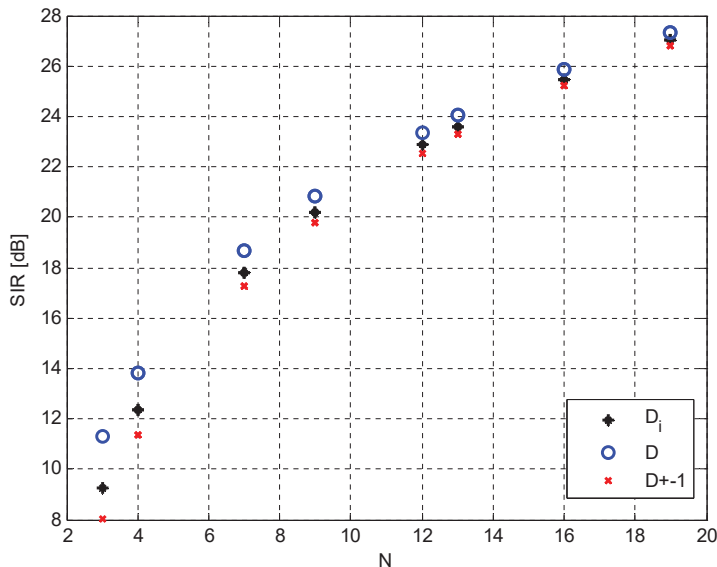
$$SIR \approx \frac{R^{-\gamma}}{2(D-R)^{-\gamma} + 2(D+R)^{-\gamma} + 2D^{-\gamma}}$$

$$= \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}$$

Again, $Q = \frac{D}{R} = \sqrt{3N}$.

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Comparison



$$SIR \approx \frac{1}{\sum_{i=1}^6 \left(\frac{D_i}{R}\right)^{-\gamma}}$$

$$SIR \approx \frac{1}{6} Q^\gamma$$

$$SIR \approx \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}$$

$$Q = \frac{D}{R}$$

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SIR Threshold

[Schwartz, 2005, p 64]

- The SIR should be greater than a specified threshold for proper signal operation.
- In the 1G **AMPS** system, designed for **voice** calls, the threshold for acceptable voice quality is SIR equal to **18 dB**.
- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.
- For the **GSM** system, a range of **7–12 dB**, depending on the study done, is suggested as the appropriate threshold.
- The probability of error in a digital system depends on the choice of this threshold as well.

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